Hadrons in vacuum and at nonzero temperature and density

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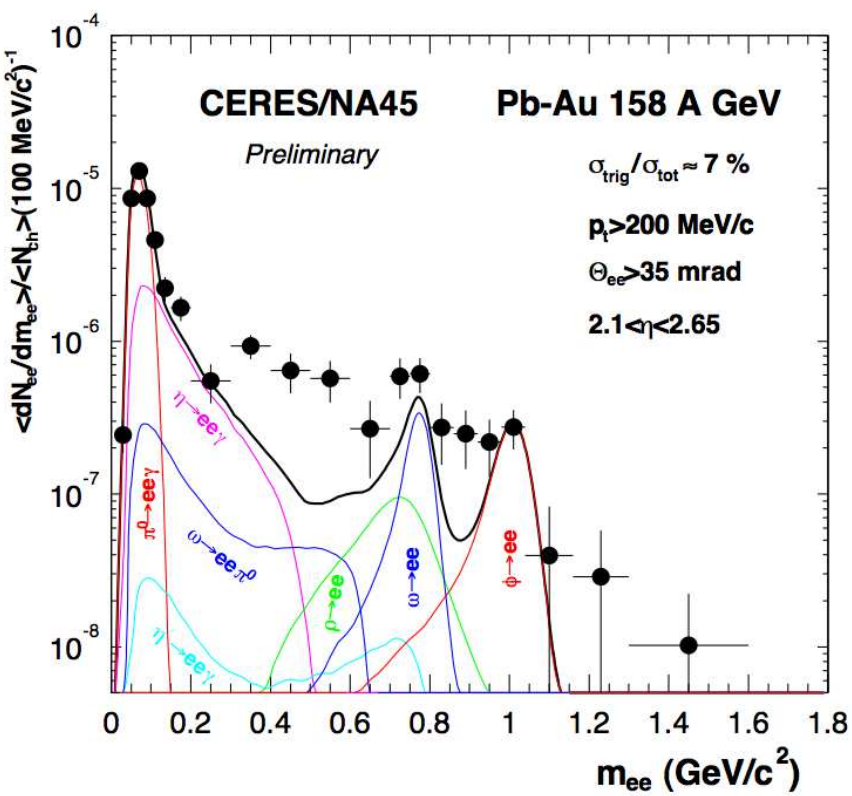
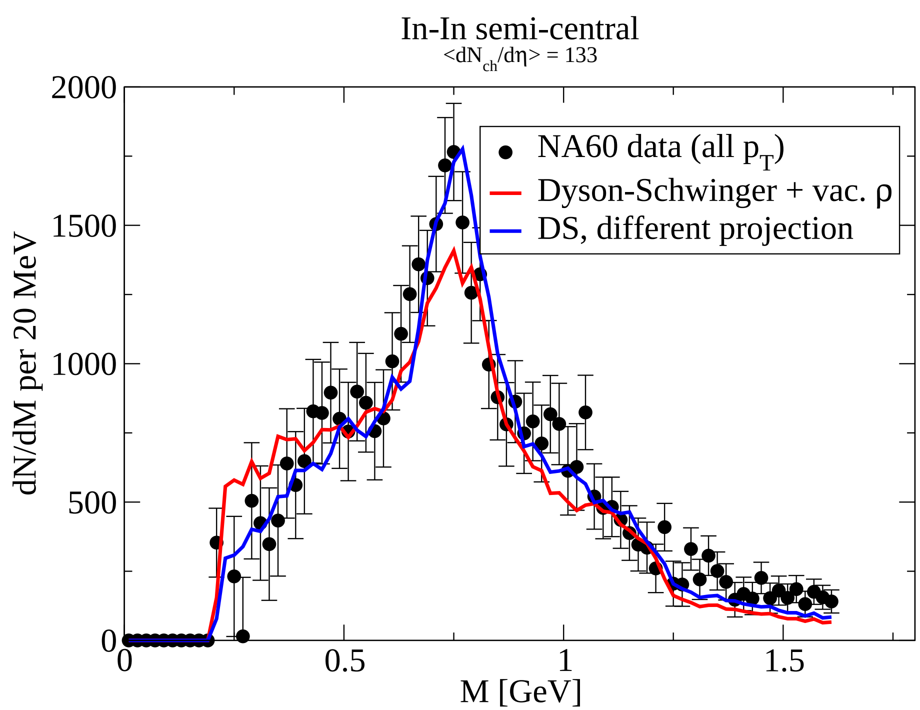
Motivation (I)

Dileptons carry information from hot and dense stages of heavy-ion collisions:



CERES/NA45 collaboration



NA60 collaboration

(fig. courtesy of Thorsten Renk)

$\Longrightarrow$ **learn about chiral symmetry restoration in hot and dense hadronic matter!**

see R. Rapp, J. Wambach, Adv. Nucl. Phys. 25 (2000) 1

The chiral effective model

Chiral symmetry of QCD: global $U(N_f)_r \times U(N_f)_\ell$ symmetry (classically)

$\Longrightarrow$ **spontaneously broken in vacuum by nonzero quark condensate $\langle \bar{q} q \rangle \neq 0$**

$\Longrightarrow$ **restored at nonzero temperature T and chemical potential μ**

$\Longrightarrow$ **degeneracy of hadronic chiral partners in the chirally restored phase**

$\Longrightarrow$ **for this application: chiral symmetry must be linearly realized**

$\Longrightarrow$ **Linear sigma model**

Disclaimer: No attempt to fit precision data for hadron vacuum phenomenology!

(No attempt to compete with chiral perturbation theory)

Nevertheless: achieve reasonable description of hadron vacuum phenomenology!

Moreover: strong statement on the nature of the scalar mesons!

scalar-meson puzzle: too many scalar states to fit into a $q\bar{q}$ meson nonet

$$f_0(500),\ f_0(980),\ f_0(1370),\ f_0(1500),\ f_0(1710)$$

$\Longrightarrow$ **Jaffe's conjecture:** R.L. Jaffe, PRD 15 (1977) 267, 281

two scalar $[qq][\bar{q}\bar{q}]$ tetraquark states mix with two scalar $q\bar{q}$ meson states

$\Longrightarrow$ **fifth scalar meson could be due to mixing with glueball**

Scalar and pseudoscalar mesons

$$\mathcal{L}_S = \mathrm{Tr}\left(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2\, \Phi^\dagger \Phi\right) - \lambda_1 \left[\mathrm{Tr}\left(\Phi^\dagger \Phi\right)\right]^2 - \lambda_2 \mathrm{Tr}\left(\Phi^\dagger \Phi\right)^2 + c\left(\det\Phi - \det\Phi^\dagger\right)^2 + \mathrm{Tr}\left[H\left(\Phi + \Phi^\dagger\right)\right] + \mathrm{Tr}\left[E\,\Phi^\dagger \Phi\right]$$

$\Phi \in (N_f^*, N_f)$ $\Longrightarrow$ $\Phi \equiv \phi_a T_a$, T_a generators of $U(N_f)$, $\phi_a \equiv \sigma_a + i\pi_a$,

$H \equiv h_a C_a$, $E \equiv \epsilon_a C_a$, $C_a \equiv T_a$, $a = 3, 8$

$\Longrightarrow$ H, E account for different non-zero quark masses

$h_a = \epsilon_a = c = 0$, $m^2 > 0$: $U(N_f)_r \times U(N_f)_\ell$ symmetry

$h_a = \epsilon_a = c = 0$, $m^2 < 0$: v.e.v. $\langle \Phi \rangle = \phi\, N_f\, T_0$, $\phi \equiv \langle \sigma_0 \rangle > 0$

Spontaneous symmetry breaking (SSB):

$U(N_f)_r \times U(N_f)_\ell \to U(N_f)_V$ $(V \equiv \ell + r)$

$h_a = \epsilon_a = 0$, $c \neq 0$: $U(1)_A$ anomaly $(A \equiv \ell - r)$

Explicit symmetry breaking (ESB):

$U(N_f)_r \times U(N_f)_\ell \to SU(N_f)_r \times SU(N_f)_\ell \times U(1)_V$

$m^2 < 0$: SSB: $SU(N_f)_r \times SU(N_f)_\ell \to SU(N_f)_V$

$\dim[SU(N_f)_r \times SU(N_f)_\ell / SU(N_f)_V] = N_f^2 - 1$

$\Longrightarrow$ $N_f^2 - 1$ Goldstone bosons $\Longrightarrow$ pseudoscalar mesons!

h_a, ϵ_a, $c \neq 0$, $m^2 < 0$: ESB $\Longrightarrow$ $N_f^2 - 1$ pseudo – Goldstone bosons

Vector and axial-vector mesons

$$\begin{aligned}
\mathcal{L}_V = & -\frac{1}{4}\,\mathrm{Tr}(\mathcal{L}^0_{\mu\nu}\mathcal{L}_0^{\mu\nu} + \mathcal{R}^0_{\mu\nu}\mathcal{R}_0^{\mu\nu}) + \mathrm{Tr}\left[\left(\frac{1}{2}\,m_1^2 + \Delta\right)\,(\mathcal{L}_\mu\mathcal{L}^\mu + \mathcal{R}_\mu\mathcal{R}^\mu)\right] \\
& + i\,\frac{g_2}{2}\,\mathrm{Tr}\left\{\mathcal{L}^0_{\mu\nu}[\mathcal{L}^\mu,\mathcal{L}^\nu] + \mathcal{R}^0_{\mu\nu}[\mathcal{R}^\mu,\mathcal{R}^\nu]\right\} \\
& + g_3\,\mathrm{Tr}\,(\mathcal{L}^\mu\mathcal{L}^\nu\mathcal{L}_\mu\mathcal{L}_\nu + \mathcal{R}^\mu\mathcal{R}^\nu\mathcal{R}_\mu\mathcal{R}_\nu) - g_4\,\mathrm{Tr}\,(\mathcal{L}^\mu\mathcal{L}_\mu\mathcal{L}^\nu\mathcal{L}_\nu + \mathcal{R}^\mu\mathcal{R}_\mu\mathcal{R}^\nu\mathcal{R}_\nu) \\
& + g_5\,\mathrm{Tr}\,(\mathcal{L}^\mu\mathcal{L}_\mu)\;\mathrm{Tr}\,(\mathcal{R}^\nu\mathcal{R}_\nu) \\
& + g_6\,[\mathrm{Tr}\,(\mathcal{L}^\mu\mathcal{L}_\mu)\;\mathrm{Tr}\,(\mathcal{L}^\nu\mathcal{L}_\nu) + \mathrm{Tr}\,(\mathcal{R}^\mu\mathcal{R}_\mu)\;\mathrm{Tr}\,(\mathcal{R}^\nu\mathcal{R}_\nu)]
\end{aligned}$$

$$\mathcal{L}^0_{\mu\nu} \equiv \partial_\mu\mathcal{L}_\nu - \partial_\nu\mathcal{L}_\mu,\;\; \mathcal{R}^0_{\mu\nu} \equiv \partial_\mu\mathcal{R}_\nu - \partial_\nu\mathcal{R}_\mu,\;\; \mathcal{L}_\mu \equiv L^a_\mu T_a,\;\; \mathcal{R}_\mu \equiv R^a_\mu T_a$$

vector mesons: $V^a_\mu \equiv \frac{1}{2}\left(L^\mu_a + R^a_\mu\right)$, axial-vector mesons: $A^a_\mu \equiv \frac{1}{2}\left(L^\mu_a - R^a_\mu\right)$

$\Delta = \delta_a C_a$: accounts for different quark masses (like E)

g_3, g_4, g_5, g_6: not determined by global fit to masses and decay widths

Scalar – vector interactions

$$\mathcal{L}_{SV} = i\, g_1 \,\mathrm{Tr}\left[\partial_\mu \Phi \left(\Phi^\dagger \mathcal{L}^\mu - \mathcal{R}^\mu \Phi^\dagger\right) - \partial_\mu \Phi^\dagger \left(\mathcal{L}^\mu \Phi - \Phi \mathcal{R}^\mu\right)\right] + \frac{h_1}{2}\,\mathrm{Tr}\left(\Phi^\dagger \Phi\right) \mathrm{Tr}\left(\mathcal{L}_\mu \mathcal{L}^\mu + \mathcal{R}_\mu \mathcal{R}^\mu\right) + (g_1^2 + h_2)\,\mathrm{Tr}\left(\Phi^\dagger \Phi \mathcal{R}_\mu \mathcal{R}^\mu + \Phi \Phi^\dagger \mathcal{L}_\mu \mathcal{L}^\mu\right) - 2(g_1^2 - h_3)\,\mathrm{Tr}\left(\Phi^\dagger \mathcal{L}_\mu \Phi \mathcal{R}^\mu\right)$$

SSB:

- induces mass splitting, e.g. $m_{a_1}^2 - m_\rho^2 = (g_1^2 - h_3)\phi_N^2$
- induces bilinear terms, e.g. $\sim g_1\, d_{abc}\, \phi_a\, A_b^\mu\, \partial_\mu \pi_c$:

$\Longrightarrow$ eliminate by shift, e.g. $A_a^\mu \rightarrow A_a^\mu + w_{a_1}(\phi_N)\, \partial^\mu \pi_a \; , \; a = 1, 2, 3,$

$$w_{a_1}(\phi_N) \equiv \frac{g_1 \phi_N}{m_{a_1}^2}$$

$\Longrightarrow$ wave function renormalization of scalar and pseudoscalar fields, e.g.

$$\pi_a \rightarrow Z_\pi\, \pi_a \; , \quad Z_\pi^2 \equiv \left(1 - \frac{g_1^2 \phi_N^2}{m_{a_1}^2}\right)^{-1} \qquad (\text{ KSFR} : Z_\pi \equiv \sqrt{2}\ )$$

$\Longrightarrow$ v.e.v. $\phi_N \equiv Z_\pi\, f_\pi$

$\Longrightarrow$ complete meson Lagrangian $\boxed{\mathcal{L}_M = \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_{SV}}$

Vacuum phenomenology: Global fit for $N_f = 3$ (I)

$N_f = 3$ $\Longrightarrow$ two scalar-isoscalar mesons f_0^L, f_0^H (combinations of $\bar{q}q$ and $\bar{s}s$)
$\Longrightarrow$ all (pseudo-)scalar masses and decay widths except those of f_0^L, f_0^H determined by linear combination of m^2, λ_1 and of m_1^2, h_1

Since nature of scalar-isoscalar mesons (quarkonium, glueball, or tetraquark?) is unclear

$\Longrightarrow$ at first omit scalar-isoscalar mesons from the fit

$\Longrightarrow$ perform χ^2–fit of $m^2, \lambda_2, c, h_0, h_8, m_1^2, \delta_S, g_1, g_2, h_2, h_3$ (11 parameters) to 21 experimental quantities

D. Parganlija, F. Giacosa, P. Kovacs, Gy. Wolf, DHR, PRD 87 (2013) 014011

Constraints: (i) no isospin violation

$\Longrightarrow$ experimental error = max(PDG error , 5%)

(ii) $m^2 < 0$ (SSB)

(iii) $\lambda_2 > 0$, $\lambda_1 > -\lambda_2/2$ (boundedness of potential)

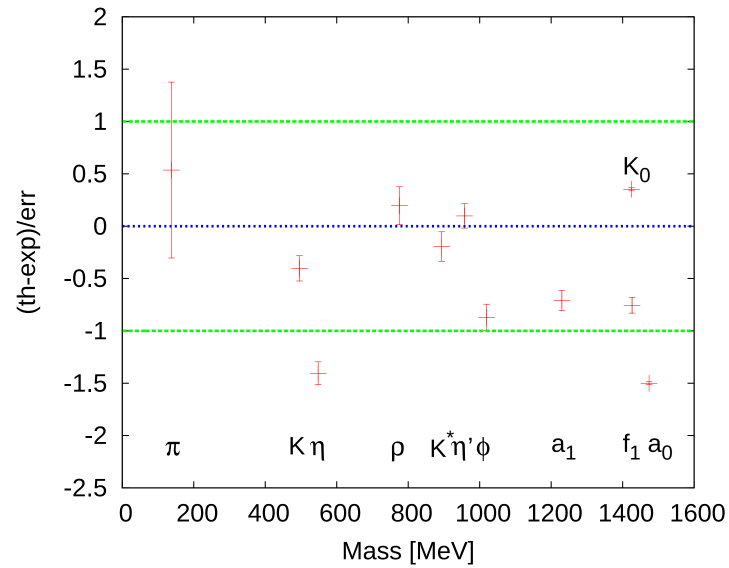
(iv) $m_1 \geq 0$ (boundedness of potential)

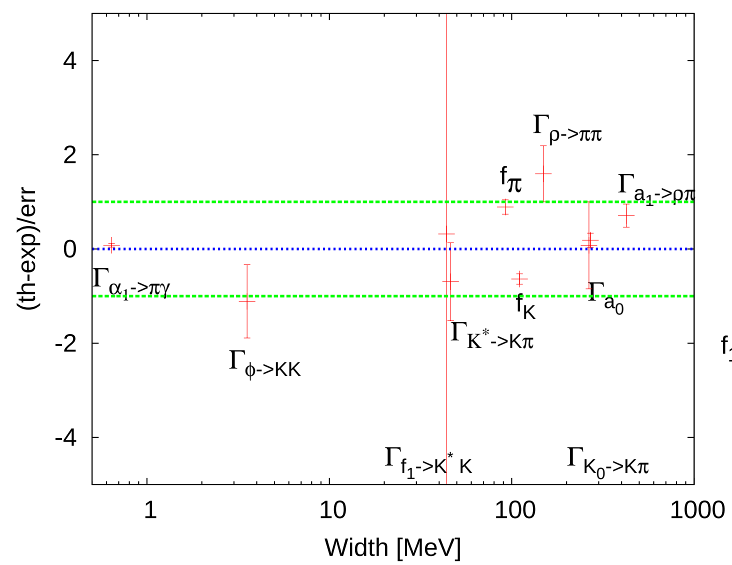
(v) $m_1 \leq m_\rho$ (SSB increases mass of vector mesons)

Vacuum phenomenology: Global fit for $N_f = 3$ (II)

Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80

accuracy of fit: $\chi^2/\text{d.o.f.} \simeq 1.23$





Vacuum phenomenology: Global fit for $N_f = 3$ (III)

large–N_c suppressed parameters $\lambda_1 = h_1 \equiv 0$:

$\Longrightarrow$ prediction for the masses of the isoscalar-scalar states:

$m_{f_0^L} = 1362.7$ MeV, $m_{f_0^H} = 1531.7$ MeV

$\Longrightarrow$ masses are in the range of the heavy scalar states:

$m_{f_0(1370)} = (1350 \pm 150)$ MeV, $m_{f_0(1500)} = (1505 \pm 75)$ MeV,

$m_{f_0(1710)} = 1720 \pm 86$ MeV

$\Longrightarrow$ mass of f_0^L close to mass of $f_0(1370)$

$\Longrightarrow$ mass of f_0^H close to $f_0(1500)$

$\Longrightarrow$ $f_0(1370)\,,\ f_0(1500)$ appear to be (predominantly) $\bar{q}q$–states

$\Longrightarrow$ chiral partners of π, η'!

$\Longrightarrow$ light scalar states $f_0(500)$, $f_0(980)$ could be (predominantly) $[qq][\bar{q}\bar{q}]$–states

(see, however, W. Heupel, G. Eichmann, C.S. Fischer, PLB 718 (2012) 545

$\Longrightarrow$ light scalars have a dominant $(\bar{q}q)(\bar{q}q)$ component!)

Incorporating the scalar glueball (I)

Another confirmation of the (predominantly) $q\bar{q}$ assignment for the heavy scalar mesons: $\Longrightarrow$ coupling to the glueball/dilaton field!

$N_f = 2$: S. Janowski, D. Parganlija, F. Giacosa, DHR, PRD 84 (2011) 054007

$N_f = 3$: S. Janowski, F. Giacosa, DHR, arXiv:1408.49.21 [hep-ph]

- dilatation symmetry $\Longrightarrow$ dynamical generation of tree-level meson mass parameters through glueball field G: $m^2 \to m^2 \left(\frac{G}{G_0}\right)^2$, $\quad m_1^2 \to m_1^2 \left(\frac{G}{G_0}\right)^2$
- add glueball Lagrangian:

$$\mathcal{L}_G = \frac{1}{2}\,(\partial_\mu G)^2 - \frac{1}{4}\,\frac{m_G^2}{\Lambda^2}\,G^4 \left(\ln\left|\frac{G}{\Lambda}\right| - \frac{1}{4}\right)$$

$\Lambda \sim$ gluon condensate $\langle G^a_{\mu\nu} G^{\mu\nu}_a \rangle$

$\Longrightarrow \quad \mathcal{L}_M \longrightarrow \mathcal{L}_M + \mathcal{L}_G$

- shift σ_N, σ_S, and G by their v.e.v.'s, $\sigma_{N,S} \to \sigma_{N,S} + \phi_{N,S}\,,\ G \to G + G_0$

$\Longrightarrow$ v.e.v. G_0 given by $\quad -\dfrac{m^2\,\Lambda^2}{m_G^2}\,(\phi_N^2 + \phi_S^2) = G_0^4\,\ln\left|\dfrac{G_0}{\Lambda}\right|$

$\Longrightarrow$ glueball mass given by $\quad M_G^2 = \frac{m^2}{G_0^2}\,(\phi_N^2 + \phi_S^2) + m_G^2\,\frac{G_0^2}{\Lambda^2}\left(1 + 3\ln\left|\frac{G_0}{\Lambda}\right|\right)$

$\Longrightarrow$ diagonalize mass matrix

$$M \equiv \begin{pmatrix} m^2_{\sigma_N} & 2\,\lambda_1 \phi_N \phi_S & 2\,m^2 \phi_N G_0^{-1} \\ 2\,\lambda_1 \phi_N \phi_S & m^2_{\sigma_S} & 2\,m^2 \phi_S G_0^{-1} \\ 2\,m^2 \phi_N G_0^{-1} & 2 m^2 \phi_S G_0^{-1} & M_G^2 \end{pmatrix}$$

Incorporating the scalar glueball (II)

⟹ χ^2 fit of Λ, λ_1, h_1, m_G, ϵ_S to the following experimental quantities:

Quantity	Our Value [MeV]	Experiment [MeV]
$M_{f_0(1370)}$	1444	1350 ± 150
$M_{f_0(1500)}$	1534	1505 ± 6
$M_{f_0(1710)}$	1750	1720 ± 6
$f_0(1370) \to \pi\pi$	423.6	325 ± 100
$f_0(1500) \to \pi\pi$	39.2	38.04 ± 4.95
$f_0(1500) \to K\bar{K}$	9.1	9.37 ± 1.69
$f_0(1710) \to \pi\pi$	28.3	29.3 ± 6.5
$f_0(1710) \to K\bar{K}$	73.4	71.4 ± 29.1

$\chi^2/\text{d.o.f.} \simeq 0.35$

⟹ $O(3)-$mixing matrix $O \equiv \begin{pmatrix} -0.91 & 0.24 & -0.33 \\ 0.30 & 0.94 & -0.17 \\ -0.27 & 0.26 & 0.93 \end{pmatrix}$

$f_0(1370)$: $83\%\,\sigma_N$ $6\%\,\sigma_S$ $11\%\,G$

⟹ $f_0(1500)$: $9\%\,\sigma_N$ $88\%\,\sigma_S$ $3\%\,G$

$f_0(1710)$: $8\%\,\sigma_N$ $6\%\,\sigma_S$ $86\%\,G$

Note: demanding dilatation symmetry of full effective model

⟹ analyticity prohibits operators with naive scaling dimension higher than 4 in Φ, $\mathcal{L}^\mu$, $\mathcal{R}^\mu$ (would require inverse powers of dilaton field)

⟹ effective model is complete!

Extension to $N_f = 4$

Fit of 3(!) additional parameters from the charm sector:

Observable	Our Value [MeV]	Exp. Value [MeV]
$m_{D_{s1}}$	2500.54	2535.12 ± 0.13
$m_{D_s^*}$	2188.33	2112.3 ±0.5
m_{D^*}	2154.58	2010.28 ± 0.13
$m_{D^{*0}}$	2154.58	2006.98 ± 0.15
m_{D_1}	2447.92	2421.3 ± 0.6
$m_{\chi_{c1}}$	3282.32	3510.66 ± 0.07
$m_{\chi_{c0}}$	3160.21	3414.75 ± 0.31
$m_{J/\psi}$	2911.3	3096.916 ± 0.011
m_{D_0}	1882.28	1864.86 ± 0.13
m_{η_c}	2490.55	2981 ± 1.1
$m_{D_0^*}$	2416.08	2403 ± 14 ± 35
m_D	1882.28	1869.62 ± 0.15
$m_{D_{s0}^*}$	2470.19	2317.8 ± 0.6
m_{D_s}	1900.39	1968.49 ± 0.32
$m_{D_0^{*0}}$	2416.08	2318 ± 29
$\Gamma_{D_1^0 \to \overline{D}^{*0}\pi^0}$	8.889	-
$\Gamma_{D_1^0 \to D^{*+}\pi^-}$	17.778	seen
$\Gamma_{D_1^+ \to D^{*0}\pi^+}$	17.778	-
$\Gamma_{D_1^+ \to D^{*+}\pi^0}$	8.88	-
$\Gamma_{D^{*0} \to D^0\pi^0}$	0.0295	<1.29
$\Gamma_{D^{*0} \to D\pi}$	0.09136	<2.1
$\Gamma_{D^{*0} \to D^+\pi^-}$	0.061	-
$\Gamma_{D^{*+} \to D^+\pi^0}$	28.1447	29.5 ± 8
$\Gamma_{D^{*+} \to D^0\pi^+}$	57.726	65 ± 17
$\Gamma_{D_0^{*+} \to D^0\pi^+}$	1.467	seen
$\Gamma_{D_0^{*+} \to D^+\pi^0}$	0.733	-
$\Gamma_{D_0^{*0} \to D^+\pi^-}$	4.159	seen
$\Gamma_{D_0^{*0} \to D^0\pi^0}$	2.079	-
$\Gamma_{D_1^0 \to \overline{D}^0\pi^+\pi^-}$	0.399	seen
$\Gamma_{D_1 \to D\pi\pi}$	0.608	-

Decay Channel	Our Value [MeV]	Exp. Value [MeV]
$\Gamma_{\chi_{c0} \to \overline{K}_0^* K_0^*}$	0.058	0.010
$\Gamma_{\chi_{c0} \to K^- K^+}$	0.001	0.063
$\Gamma_{\chi_{c0} \to \pi\pi}$	0.083	0.0884
$\Gamma_{\chi_{c0} \to a_0 a_0}$	0.080	-
$\Gamma_{\chi_{c0} \to k_1^0 K_1^0}$	0.003	-
$\Gamma_{\chi_{c0} \to \overline{K}^{*0} K^{*0}}$	0.0167	0.01768
$\Gamma_{\chi_{c0} \to \eta\eta}$	0.37	0.37
$\Gamma_{\chi_{c0} \to \eta'\eta'}$	14.09	0.021
$\Gamma_{\chi_{c0} \to \eta\eta'}$	4.839	<0.0025
$\Gamma_{\chi_{c0} \to ww}$	0.031	0.019
$\Gamma_{\chi_{c0} \to k_1^+ K^-}$	0.0669	0.066
$\Gamma_{\chi_{c0} \to K^* K_0^*}$	0.00006	-
$\Gamma_{\chi_{c0} \to \rho_0\rho_0}$	0.01606	-
$\Gamma_{\chi_{c0} \to \sigma_1\sigma_1}$	0.032	<0.0029
$\Gamma_{\chi_{c0} \to K_0^* K\eta}$	2.66	-
$\Gamma_{\chi_{c0} \to K_0^* K\eta'}$	6.47	-
$\Gamma_{\chi_{c0} \to \sigma_1\eta\eta}$	0.719	-
$\Gamma_{\chi_{c0} \to \sigma_2\eta\eta}$	0.693	-
$\Gamma_{\chi_{c0} \to \sigma_1\eta'\eta'}$	0.911	-
$\Gamma_{\chi_{c0} \to \sigma_1\eta\eta'}$	1.747	-
$\Gamma_{\chi_{c0} \to \sigma_2\eta\eta'}$	0.8116	-
$\Gamma_{\chi_{c0} \to \sigma_2\eta'\eta'}$	0.4148	-

$\chi^2/\mathrm{d.o.f.} \simeq 0.377$

W.I. Eshraim, F. Giacosa, DHR, arXiv:1405.5861[hep-ph]

Electroweak interactions

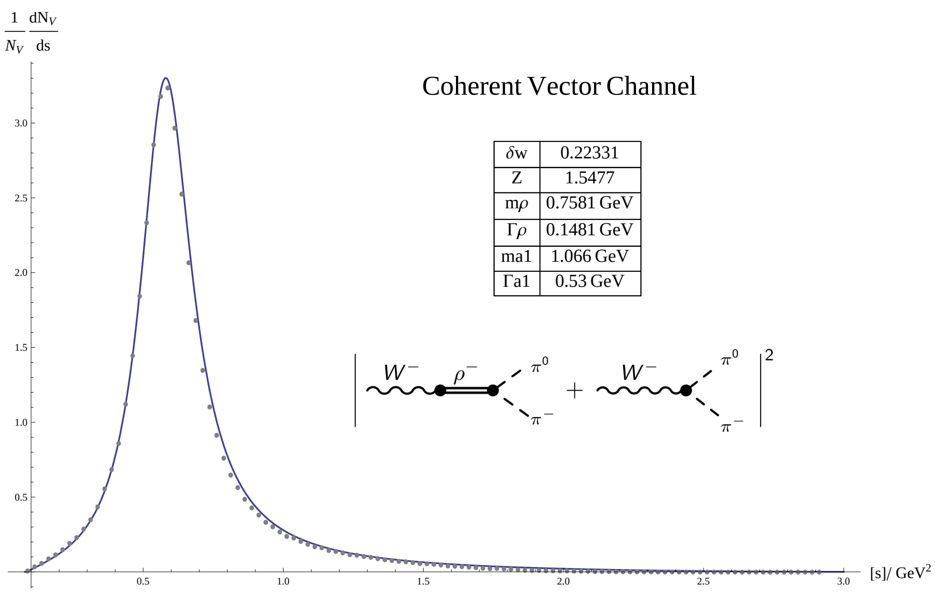
A. Habersetzer, F. Giacosa, DHR, in preparation

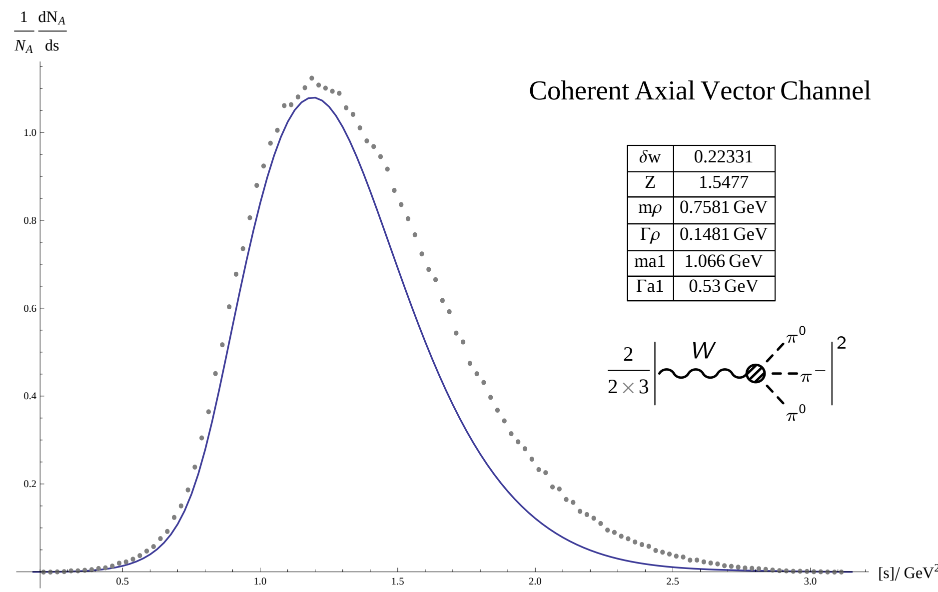
$$\partial^\mu \Phi \longrightarrow D^\mu \Phi \equiv \partial^\mu \Phi - i\, e\, A^\mu\, [T_3, \Phi] - i\, g \cos\theta_C\, (W_1^\mu T_1 + W_2^\mu T_2)\Phi$$
$$- i\, g \cos\theta_W\, (Z^\mu T_3 \Phi + \tan^2\theta_W\, \Phi T_3 Z^\mu)$$

$$\mathcal{L}_0^{\mu\nu} \longrightarrow \mathcal{L}^{\mu\nu} \equiv \partial^\mu \mathcal{L}^\nu - i\, e\, A^\mu [T_3, \mathcal{L}^\nu] - i\, g [W_1^\mu T_1 + W_2^\mu T_2, \mathcal{L}^\nu] - ig \cos\theta_W Z^\mu [T_3, \mathcal{L}^\nu]$$
$$- \partial^\nu \mathcal{L}^\mu + i\, e\, A^\nu [T_3, \mathcal{L}^\mu] + i\, g [W_1^\nu T_1 + W_2^\nu T_2, \mathcal{L}^\mu] + ig \cos\theta_W Z^\nu [T_3, \mathcal{L}^\mu]$$

$$R_0^{\mu\nu} \longrightarrow \mathcal{R}^{\mu\nu} \equiv \partial^\mu \mathcal{R}^\nu - i\, e\, A^\mu [T_3, \mathcal{R}^\nu] - ig \sin\theta_W Z^\mu [T_3, \mathcal{R}^\nu]$$
$$- \partial^\nu \mathcal{R}^\mu + i\, e\, A^\nu [T_3, \mathcal{R}^\mu] + ig \sin\theta_W Z^\nu [T_3, \mathcal{R}^\mu]$$

$$\mathcal{L}_M \longrightarrow \mathcal{L}_M + \frac{\delta}{2} g\, \cos\theta_C\, \mathrm{Tr}[W_{\mu\nu} \mathcal{L}^{\mu\nu}] + \frac{\bar{\delta}}{2} e\, \mathrm{Tr}[B_{\mu\nu} \mathcal{R}^{\mu\nu}] + \frac{1}{4}\, \mathrm{Tr}[(W^{\mu\nu})^2 + (B^{\mu\nu})^2]$$





cf. M. Urban, M. Buballa, J. Wambach, NPA 697 (2002) 338

Chiral symmetry restoration at nonzero temperature (I)

S. Strüber, DHR, PRD 77 (2008) 085004

2PI effective potential:

$$U_{\text{eff}}[\phi, G_i] = V(\phi) + \frac{1}{2} \sum_i \int_K \left[\ln G_i^{-1}(K) + D_i^{-1}(K) G_i(K) - 1 \right] + V_2[\phi, G_i]$$

$V(\phi)$: classical potential, $D_i(K)$: tree-level propagators, $V_2[\phi, G_i]$: sum of 2PI vacuum diagrams

Stationarity of the effective potential: $$\frac{\partial U_{\text{eff}}}{\partial \phi} = 0 \; , \quad \frac{\delta U_{\text{eff}}}{\delta G_i} = 0$$

$\Longrightarrow$ **Dyson–Schwinger eqs. for the full propagators:**

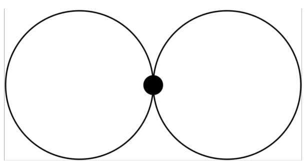
$$G_i^{-1}(K) = D_i^{-1}(K) + \Pi_i(K) \; , \quad \text{self-energy: } \Pi_i(K) = -2 \, \frac{\delta V_2}{\delta G_i(K)}$$

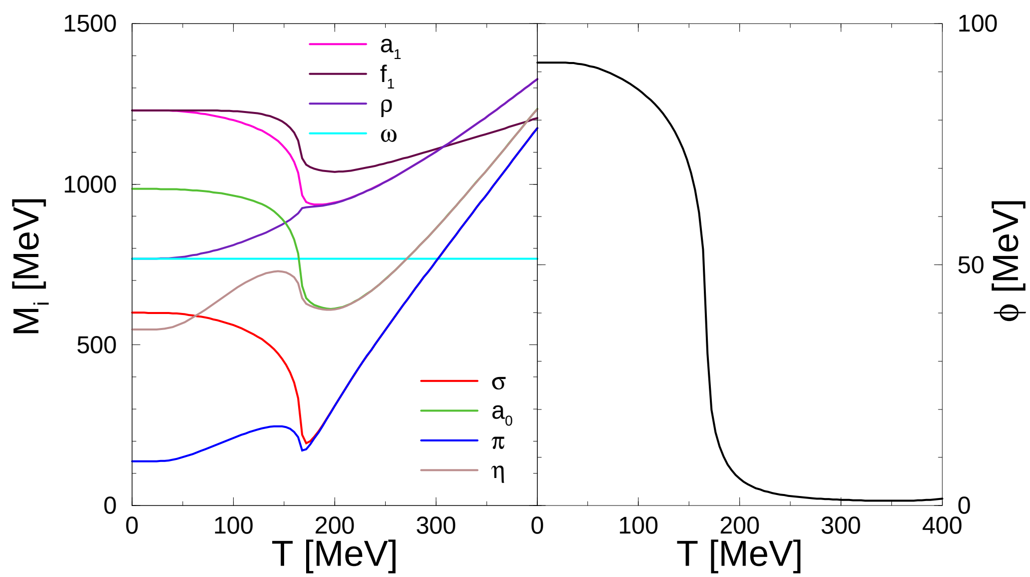
approximations:

– gauged linear sigma model

$\Longrightarrow$ $g_i \equiv g \; , \; i = 1, \ldots, 6$

– Hartree–Fock approximation:

$V_2 =$ 



Chiral symmetry restoration at nonzero temperature (II)

A. Heinz, S. Strüber, F. Giacosa, DHR, PRD 79 (2009) 037502

$O(4)-$linear sigma model without (axial-)vector mesons

but: additional light scalar–isoscalar meson

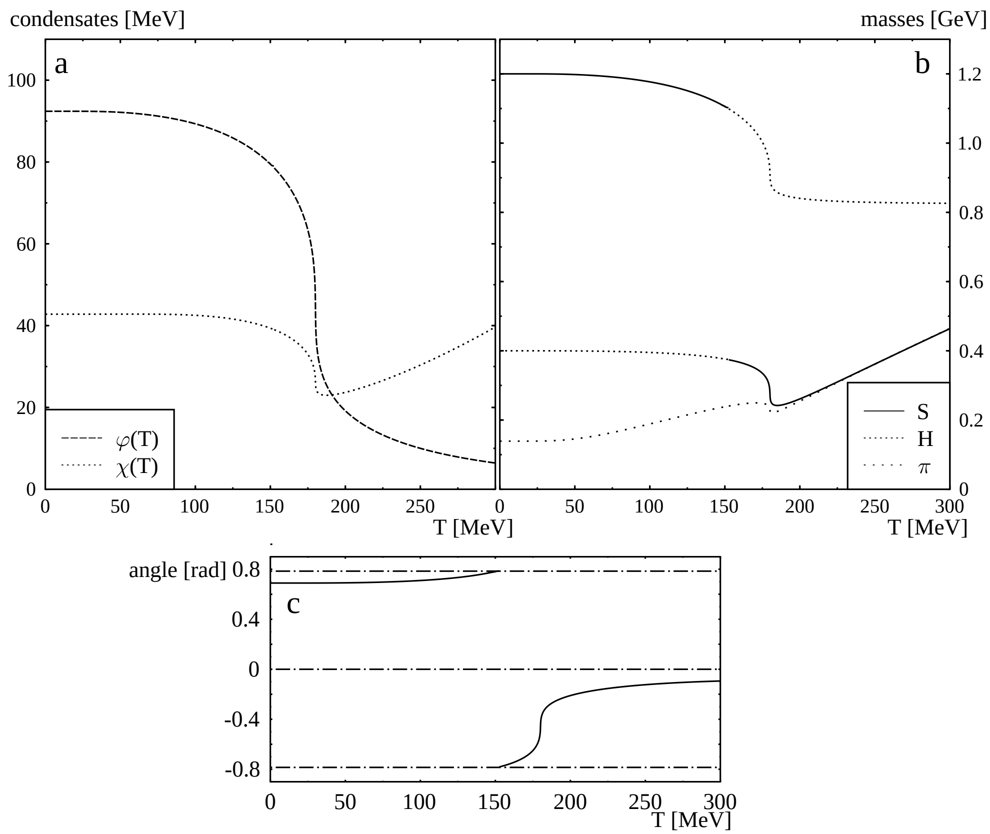
$$V(\varphi,\chi) = \frac{\lambda}{4}(\varphi^2 + \vec{\pi}^{\,2} - F^2)^2 - \varepsilon\varphi + \frac{1}{2}M_\chi^2\chi^2 - g\chi(\varphi^2 + \vec{\pi}^2)$$

$\Longrightarrow$ SSB: $\langle\varphi\rangle \equiv \varphi_0 \neq 0$,

$\Longrightarrow$ induces condensation of χ, $\langle\chi\rangle \equiv \chi_0 \neq 0$

$\Longrightarrow$ mixing of φ and χ fields

$\Longrightarrow$ diagonalize mass matrix for each T in terms of new fields S, H



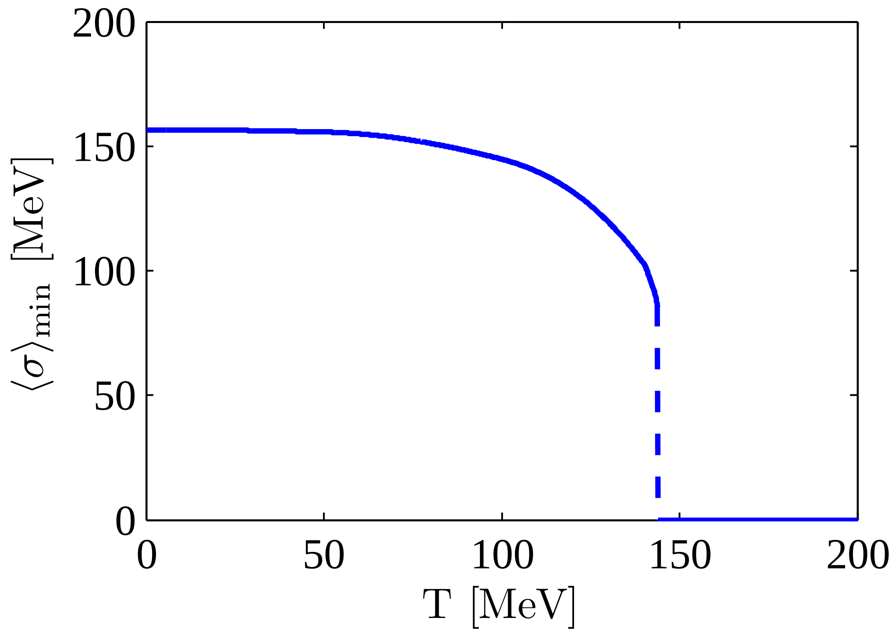
$\Longrightarrow$ 2PI effective potential in Hartree–Fock approximation

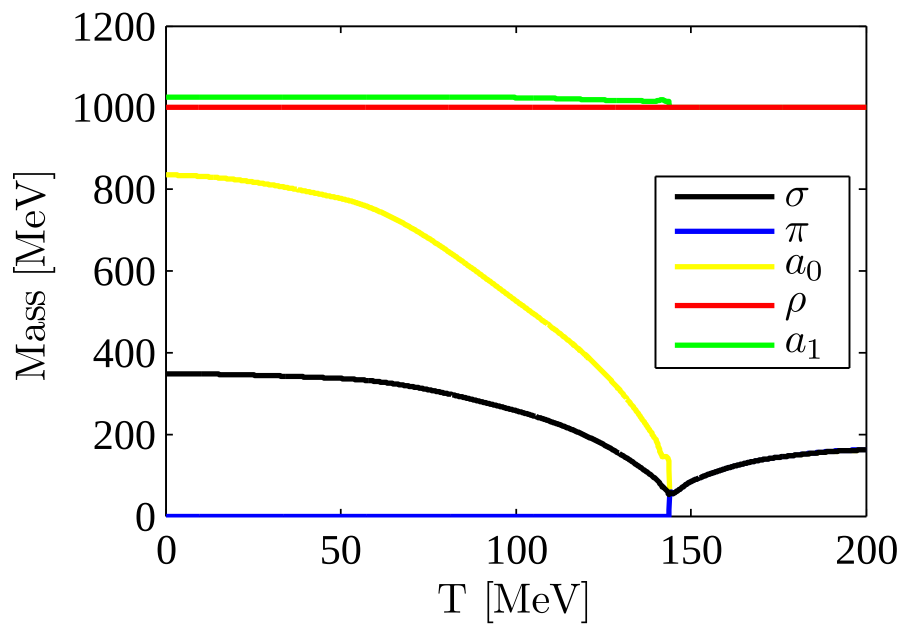
Chiral symmetry restoration at nonzero temperature (III)

J. Eser, DHR, in preparation

Effective potential within Functional Renormalization Group approach:

$$\partial_k \Gamma_k = \frac{1}{2} \, \mathrm{Tr} \left(\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right)$$





preliminary!!

Baryons and their chiral partners

Inclusion of baryons and their chiral partners (so far $N_f = 2$):

$\Longrightarrow$ **Mirror assignment:** C. DeTar and T. Kunihiro, PRD 39 (1989) 2805

$$\Psi_{1,r} \to U_r\, \Psi_{1,r}\ , \quad \Psi_{1,\ell} \to U_\ell\, \Psi_{1,\ell}\ , \quad \text{but: } \Psi_{2,r} \to U_\ell\, \Psi_{2,r}\ , \quad \Psi_{2,\ell} \to U_r\, \Psi_{2,\ell}$$

$\Longrightarrow$ **new, chirally invariant mass term:**

$$\mathcal{L}_B = \bar{\Psi}_{1,\ell}\, i\partial\!\!\!/\, \Psi_{1,\ell} + \bar{\Psi}_{1,r}\, i\partial\!\!\!/\, \Psi_{1,r} + \bar{\Psi}_{2,\ell}\, i\partial\!\!\!/\, \Psi_{2,\ell} + \bar{\Psi}_{2,r}\, i\partial\!\!\!/\, \Psi_{2,r} + m_0 \left(\bar{\Psi}_{2,\ell}\, \Psi_{1,r} - \bar{\Psi}_{2,r}\, \Psi_{1,\ell} - \bar{\Psi}_{1,\ell}\, \Psi_{2,r} + \bar{\Psi}_{1,r}\, \Psi_{2,\ell} \right)$$

Note: chiral symmetry restoration:

chiral partners become degenerate, but not necessarily massless!

$\Longrightarrow$ **m_0 models contribution from gluon condensate to baryon mass**

$\Longrightarrow$ **allows for stable nuclear matter ground state! (see below)**

Vector – baryon interactions

$$\mathcal{L}_{VB} = c_1 \left(\bar{\Psi}_{1,\ell} \not{\mathcal{L}}\, \Psi_{1,\ell} + \bar{\Psi}_{1,r} \not{\mathcal{R}}\, \Psi_{1,r}\right) + c_2 \left(\bar{\Psi}_{2,\ell} \not{\mathcal{R}}\, \Psi_{2,\ell} + \bar{\Psi}_{2,r} \not{\mathcal{L}}\, \Psi_{2,r}\right)$$

Note: in general $c_1 \neq c_2$

$\Longrightarrow$ allows to fit axial coupling constants (see below)!

Scalar – baryon interactions

Yukawa interaction:

$$\mathcal{L}_{SB} = -\hat{g}_1 \left(\bar{\Psi}_{1,\ell}\, \Phi\, \Psi_{1,r} + \bar{\Psi}_{1,r}\, \Phi^\dagger\, \Psi_{1,\ell} \right) - \hat{g}_2 \left(\bar{\Psi}_{2,r}\, \Phi\, \Psi_{2,\ell} + \bar{\Psi}_{2,\ell}\, \Phi^\dagger\, \Psi_{2,r} \right)$$

$N_f = 2$ mass eigenstates:

$$\begin{pmatrix} N \\ N^* \end{pmatrix} \equiv \begin{pmatrix} N^+ \\ N^- \end{pmatrix} = \frac{1}{\sqrt{2\cosh\delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5\, e^{-\delta/2} \\ \gamma_5\, e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} , \quad \sinh\delta = \frac{\phi}{4\, m_0} (\hat{g}_1 + \hat{g}_2)$$

$$m_\pm = \sqrt{m_0^2 + \frac{\phi^2}{16}(\hat{g}_1 + \hat{g}_2)^2} \pm \frac{\phi}{4}(\hat{g}_1 - \hat{g}_2) \quad \longrightarrow \quad m_0 \qquad (\phi \to 0)$$

axial coupling constant:

$$g_A = +\tanh\delta \left[1 - \frac{c_1 + c_2}{2\, g_1} \left(1 - \frac{1}{Z^2} \right) \right] - \frac{c_1 - c_2}{2\, g_1} \left(1 - \frac{1}{Z^2} \right)$$

$$g_A^* = -\tanh\delta \left[1 - \frac{c_1 + c_2}{2\, g_1} \left(1 - \frac{1}{Z^2} \right) \right] - \frac{c_1 - c_2}{2\, g_1} \left(1 - \frac{1}{Z^2} \right) \neq -g_A\, !$$

$\Longrightarrow$ for $c_1 \neq c_2$ compatible with $g_A \simeq 1.26$, $g_A^* \simeq 0\,!$

T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

T. Maurer, T. Burch, L.Ya. Glozman, C.B. Lang, D. Mohler, A. Schäfer, arXiv:1202.2834[hep-lat]

Vacuum phenomenology: The chiral partner of the nucleon (I)

Baryon sector ($N_f = 2$): S. Gallas, F. Giacosa, DHR, PRD 82 (2010) 014004

Determine m_0 , c_1 , c_2 , $\hat{g}_1$, $\hat{g}_2$ **through** χ^2 **fit to**

$$M_N\,,\; M_{N^*}\,,\; g_A = 1.267 \pm 0.004\,,\; g_A^*\,,\; \Gamma(N^* \to N\pi)$$

(i) Scenario A: $N = N(940),\, N^* = N(1535)$

$\Longrightarrow$ $g_A^* = 0.2 \pm 0.3$ T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

$\Gamma(N^* \to N\pi) = (67.5 \pm 23.6)$ MeV

(ii) Scenario B: $N = N(940),\, N^* = N(1650)$

$\Longrightarrow$ $g_A^* = 0.55 \pm 0.2$ T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

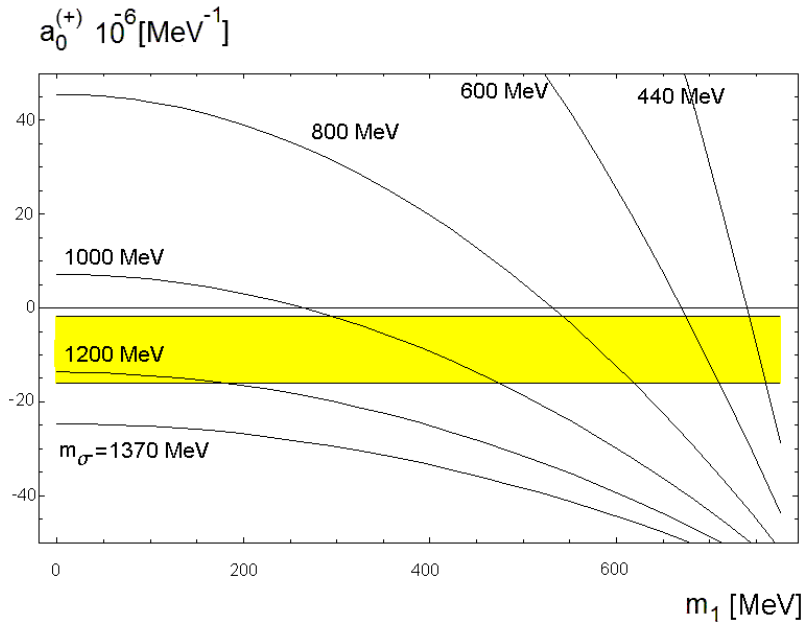
$\Gamma(N^* \to N\pi) = (128 \pm 44)$ MeV

Test validity of the two scenarios through comparison to:

- πN scattering lengths
- decay width $\Gamma(N^* \to N\eta)$

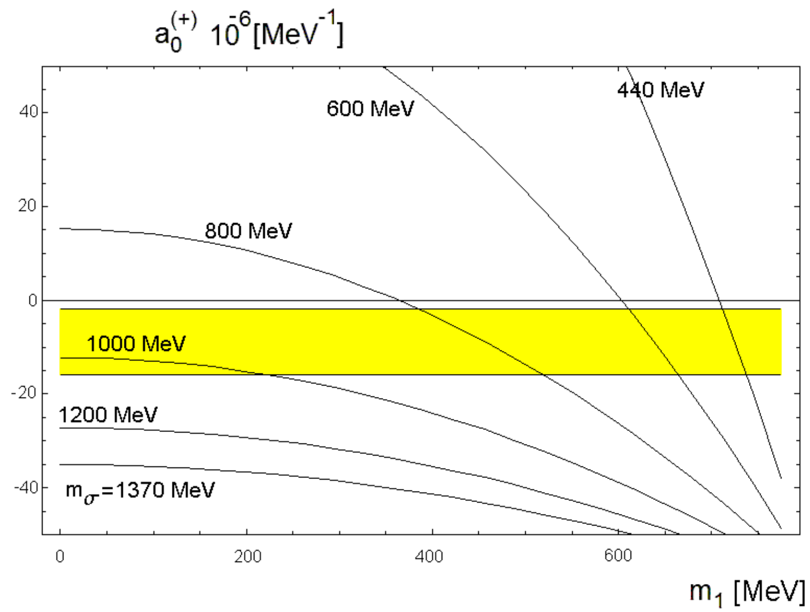
Vacuum phenomenology: The chiral partner of the nucleon (II)

πN scattering lengths $a_0^{(\pm)}$:



$m_{N^*} = 1535$ MeV

$$a_0^{(-)} = (6.04 \pm 0.63) \cdot 10^{-4}\ \text{MeV}^{-1}$$



$m_{N^*} = 1655$ MeV

$$a_0^{(-)} = (5.90 \pm 0.46) \cdot 10^{-4}\ \text{MeV}^{-1}$$

for comparison: $a_{0,\text{exp}}^{(-)} = (6.4 \pm 0.1) \cdot 10^{-4}\ \text{MeV}^{-1}$

However:

$\Gamma(N^* \to N\eta) = (10.9 \pm 3.8)$ MeV, $\quad \Gamma_{\text{exp}}(N^* \to N\eta) = (78.7 \pm 24.3)$ MeV!

$\Gamma(N^* \to N\eta) = (18.3 \pm 8.5)$ MeV, $\quad \Gamma_{\text{exp}}(N^* \to N\eta) = (10.7 \pm 6.7)$ MeV

$\Longrightarrow$ **Scenario B seems to be favored!**

Vacuum phenomenology: The chiral partner of the nucleon (III)

$\Longrightarrow$ **But then: what is the chiral partner of $N(1535)$?**

Remember L.Ya. Glozman, PRL 99 (2007) 191602:

Heavy chiral partners are closer in mass than lighter ones

$\Longrightarrow$ **Signal of chiral symmetry restoration in the QCD mass spectrum**

$\Longrightarrow$ **Could the partner of $N(1535)$ be $N(1440)$?**

Nuclear matter saturation (I)

D. Zschiesche, L. Tolos, J. Schaffner-Bielich, R.D. Pisarski, PRC 75 (2007) 055202

studied cold nuclear matter within the mirror assignment

used effective potential in mean-field approximation:

$$U_{\text{eff}}(\sigma, \omega_0) = \sum_{i=\pm} \frac{d_i}{(2\pi)^3} \int_0^{k_{F,i}} \mathrm{d}^3\vec{k}\, \left[E_i^*(k) - \mu_i^*\right] + \frac{1}{2} m^2 \sigma^2 + \frac{1}{4} \lambda \sigma^4 - h\sigma - \frac{1}{2} m_1^2 \omega_0^2 - g_4 \omega_0^4$$

d_i internal degrees of freedom of N, N^*

$k_{F,i} = \sqrt{\mu_i^{*\,2} - m_i^2}$ Fermi momentum

$E_i^*(k) = \sqrt{k^2 + m_i^2}$ single-particle energy

$\mu_i^* = \mu_i - g_\omega \omega_0$ effective chemical potential

$$m^2 = \frac{1}{2} \left(3m_\pi - m_\sigma^2\right) , \quad \lambda = \frac{m_\sigma^2 - m_\pi^2}{2\sigma} , \quad h = f_\pi m_\pi^2 ,$$

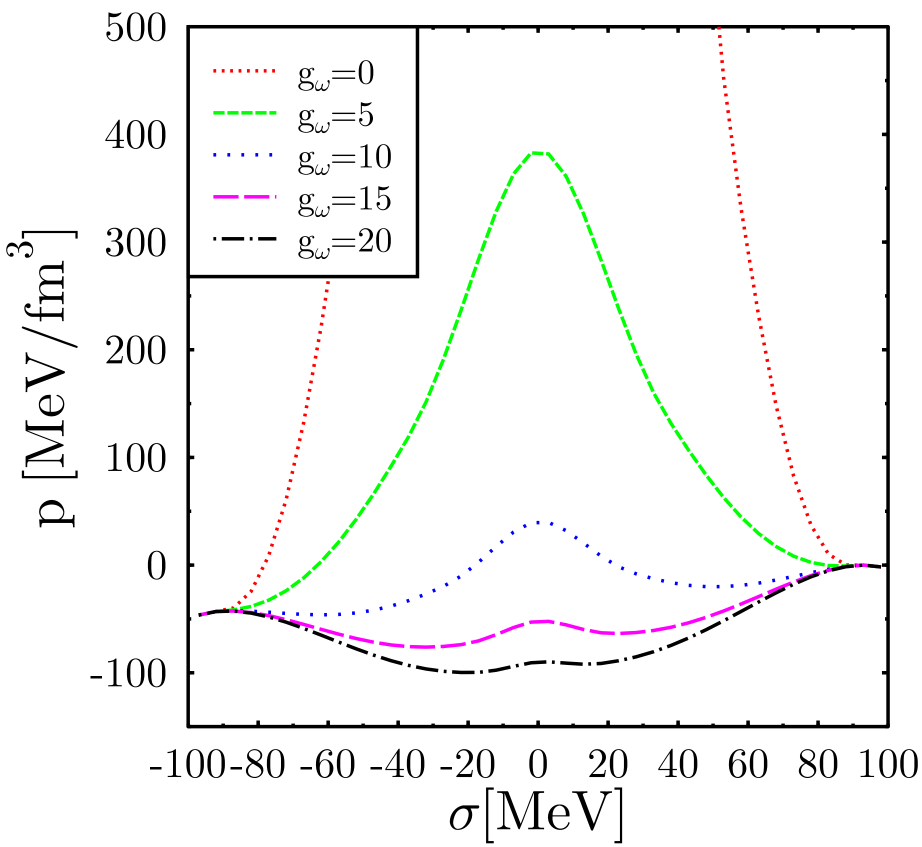
v.e.v.'s $\phi = \langle\sigma\rangle$, $\bar{\omega} = \langle\omega_0\rangle$ determined by

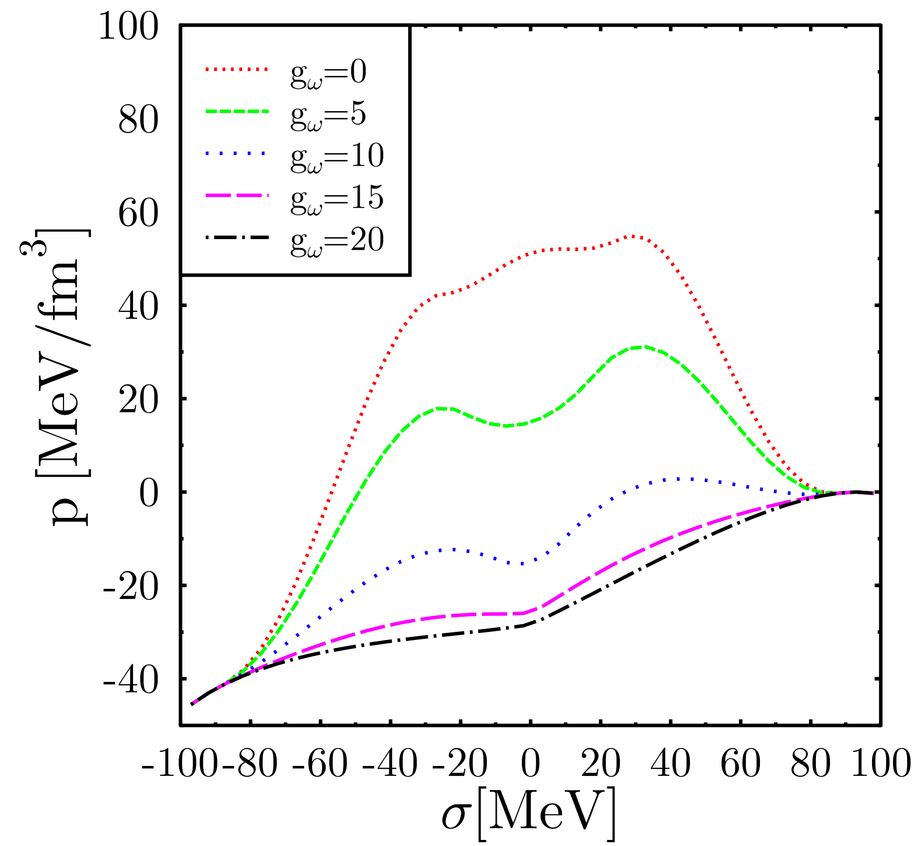
$$\left.\frac{\partial U_{\text{eff}}(\sigma, \omega_0)}{\partial\sigma}\right|_{\phi,\bar{\omega}} = \left.\frac{\partial U_{\text{eff}}(\sigma, \omega_0)}{\partial\omega_0}\right|_{\phi,\bar{\omega}} = 0$$

Nuclear matter saturation (II)

$m_0 = 0$: $\Longrightarrow$ $\nexists$ g_ω **for which nuclear matter saturates**

$m_0 > 0$: $\Longrightarrow$ $\exists$ g_ω **for which nuclear matter saturates**





$\Longrightarrow$ **ground state is either vacuum or chirally restored phase**

(**both figs.:** $\mu_B = 923$ **MeV**, $g_4 = 0$, $m_- = 1.5$ **GeV**
left: $m_\sigma = 1$ **GeV**, **right:** $m_\sigma = 400$ **MeV**)

Nuclear matter saturation (III)

$\exists$ nuclear matter ground state for:

m_- [GeV]	m_0 [MeV]	m_σ [MeV]	g_4	$m_+(n_0)/m_+$	$m_-(n_0)/m_-$	K [MeV]
1.5	790	370.63	0	0.84	0.73	510.57
1.5	790	346.59	3.8	0.83	0.72	440.51
1.2	790	318.56	0	0.86	0.79	436.41
1.2	790	302.01	3.8	0.86	0.78	374.75

$\Longrightarrow$ scalar meson too light, compressibility too large!

S. Gallas, F. Giacosa, G. Pagliara, NPA 872 (2011) 13

inclusion of tetraquark d.o.f. χ : m_0 dynamically generated, $m_0 = a\,\chi$

$\Longrightarrow$ $U_{\rm eff}(\sigma, \omega_0, \chi) = U_{\rm eff}(\sigma, \omega_0) - g\,\chi\,\sigma^2 + \frac{1}{2}\,m_\chi^2\,\chi^2$

v.e.v. $\bar{\chi} = \langle \chi \rangle$ determined by $\left.\dfrac{\partial U_{\rm eff}(\sigma, \omega_0, \chi)}{\partial \chi}\right|_{\phi, \bar{\omega}, \bar{\chi}} = 0$

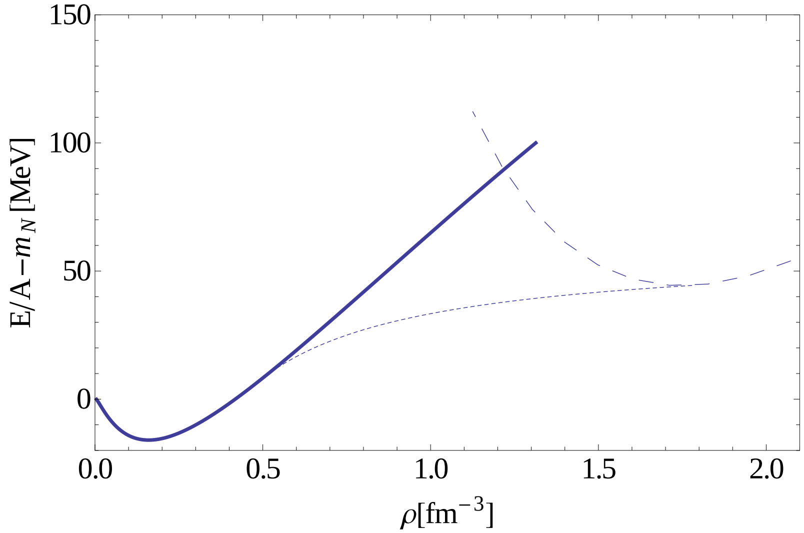
$\Longrightarrow$ nuclear matter ground state:

m_- [GeV]	m_0 [MeV]	m_σ [GeV]	g_4	m_χ [MeV]	K [MeV]
1.535	500	1.294	0	612	194

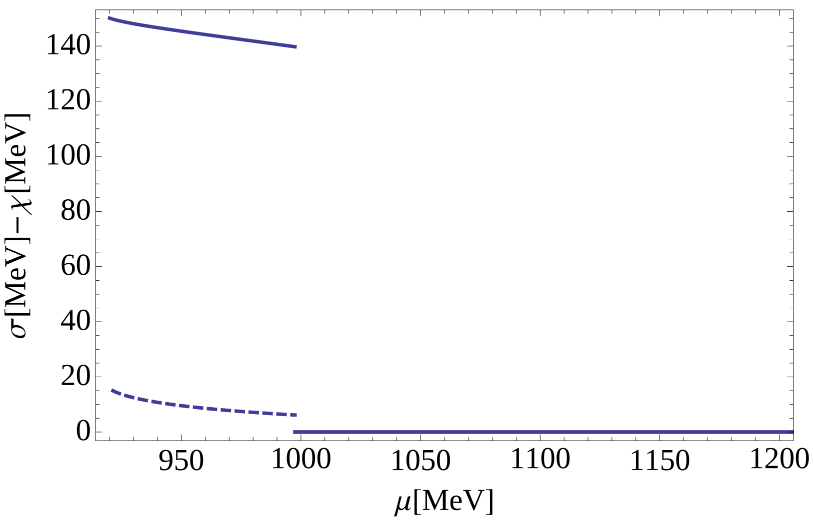
Note: fit to vacuum properties requires $m_0 = 460 \pm 130$ MeV

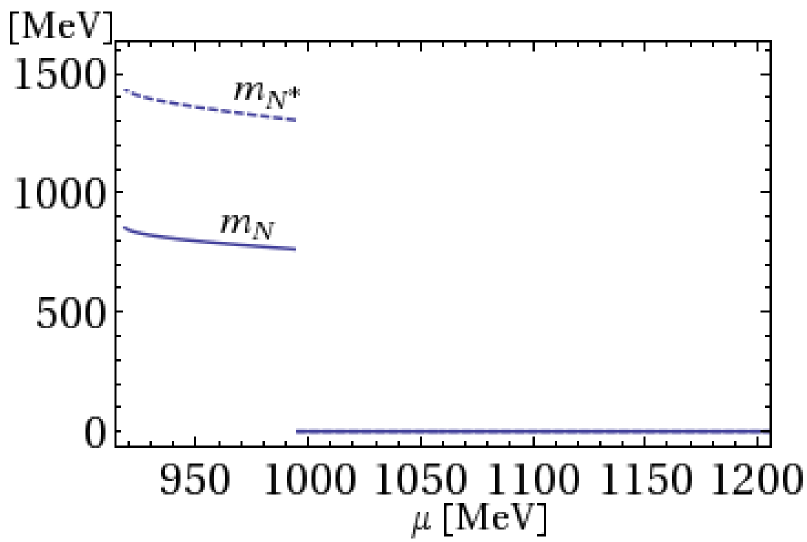
Nuclear matter at large densities

⟹ **first-order phase transition to chirally restored phase:**



S. Gallas, F. Giacosa, G. Pagliara, NPA 872 (2011) 13





Chiral density wave in nuclear matter (I)

A. Heinz, F. Giacosa, DHR, arXiv:1312.3244 [nucl-th]

retain only fields that develop a v.e.v.: $\sigma\,,\ \pi \equiv \pi^3\,,\ \omega_\mu\,,\ \chi$

$$\mathcal{L}_{\rm mes} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{1}{2}m^2(\sigma^2+\pi^2) - \frac{\lambda}{4}(\sigma^2+\pi^2)^2 + \varepsilon\sigma - \frac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)^2 + \frac{1}{2}m_\omega^2\omega_\mu^2 + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\chi^2\chi^2 + g\chi(\sigma^2+\pi^2)$$

where $m_\sigma = 1295$ MeV, $m_\omega = 782$ MeV, $m_\chi = 611$ MeV

$$\mathcal{L}_{\rm bar} = \overline{\Psi}_1 i\gamma_\mu\partial^\mu\Psi_1 + \overline{\Psi}_2 i\gamma_\mu\partial^\mu\Psi_2 - \frac{\hat{g}_1}{2}\overline{\Psi}_1(\sigma + i\gamma_5\tau^3\pi)\Psi_1 - \frac{\hat{g}_2}{2}\overline{\Psi}_2(\sigma - i\gamma_5\tau^3\pi)\Psi_2 - g_\omega\overline{\Psi}_1 i\gamma_\mu\omega^\mu\Psi_1 - g_\omega\overline{\Psi}_2 i\gamma_\mu\omega^\mu\Psi_2 - a\chi(\overline{\Psi}_2\gamma_5\Psi_1 - \overline{\Psi}_1\gamma_5\Psi_2)$$

Ansatz for chiral density wave: $\langle\sigma\rangle = \phi\cos(2fx)\,,\quad \langle\pi\rangle = \phi\sin(2fx)$

$\Longrightarrow$ **coordinate dep. in $\mathcal{L}_{\rm bar}$ can be transformed into momentum dep.:**

$$\Psi_1 \to \exp[-i\gamma_5\tau_3 fx]\Psi_1\,,\quad \Psi_2 \to \exp[+i\gamma_5\tau_3 fx]\Psi_2$$

$\Longrightarrow$ **effective potential:**

$$U_{\rm eff}(\phi,\bar{\chi},\bar{\omega}_0,f) = 2f^2\phi^2 + \frac{\lambda}{4}\phi^4 - \frac{1}{2}m^2\phi^2 - \varepsilon\phi\cos(2fx) - \frac{1}{2}m_\omega^2\bar{\omega}_0^2 + \frac{1}{2}m_\chi^2\bar{\chi}^2 - g\bar{\chi}\phi^2 + 2\sum_{k=1}^{4}\int\frac{{\rm d}^3\vec{p}}{(2\pi)^3}[E_k(p)-\mu^*]\Theta[\mu^*-E_k(p)]$$

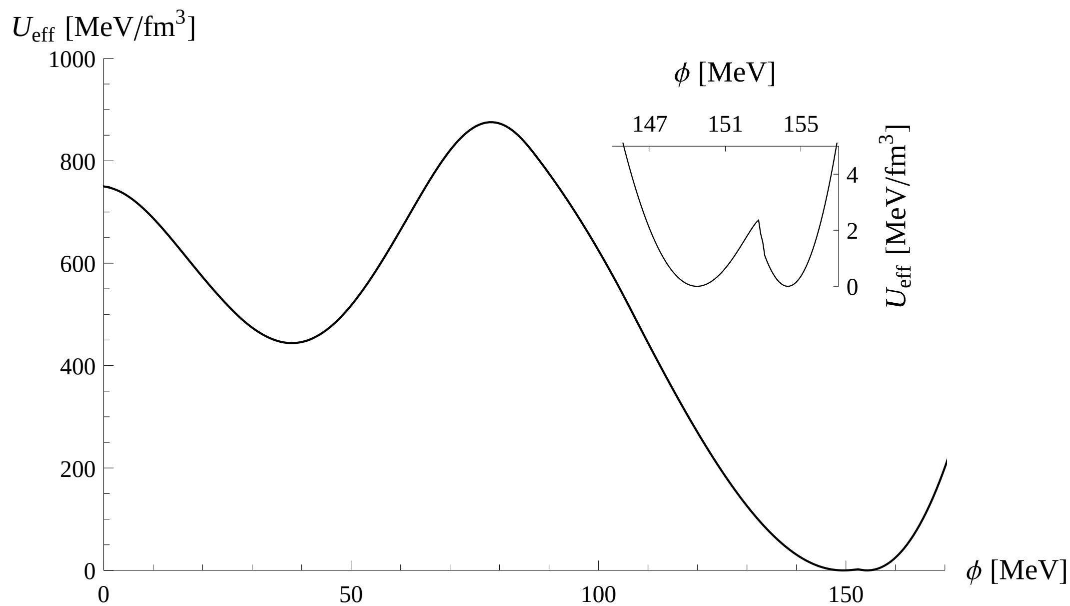
where $\mu^* = \mu - g_\omega\bar{\omega}_0\,,\quad E_k(p) = \sqrt{p^2 + \bar{m}_k(p_x)^2}$

Chiral density wave in nuclear matter (II)

ground state is obtained by minimizing $U_{\rm eff}$ with respect to meson mean fields:

$$0 \stackrel{!}{=} \frac{\partial U_{\rm eff}}{\partial \phi}, \; 0 \stackrel{!}{=} \frac{\partial U_{\rm eff}}{\partial \bar{\chi}}, \; 0 \stackrel{!}{=} \frac{\partial U_{\rm eff}}{\partial \bar{\omega}_0}, \; 0 \stackrel{!}{=} \frac{\partial U_{\rm eff}}{\partial f}$$

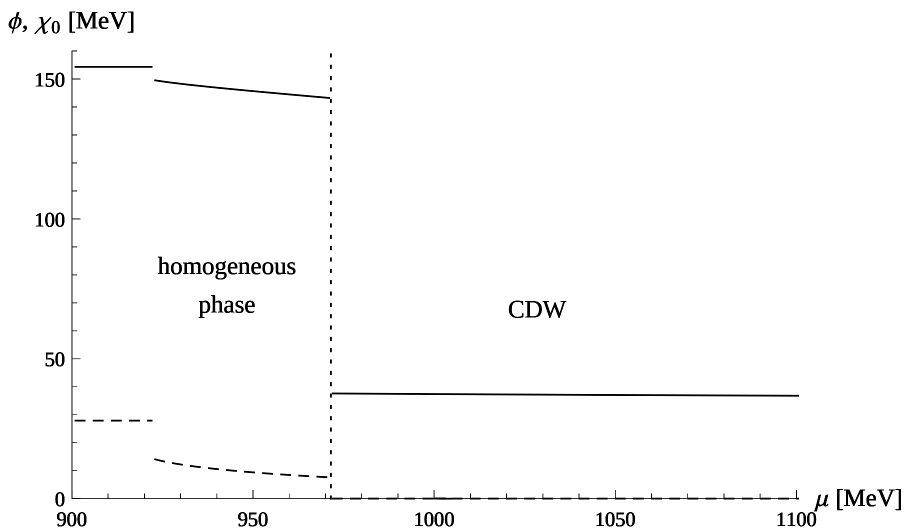
$\Longrightarrow$ $U_{\rm eff}$ at $\mu = 923$ MeV

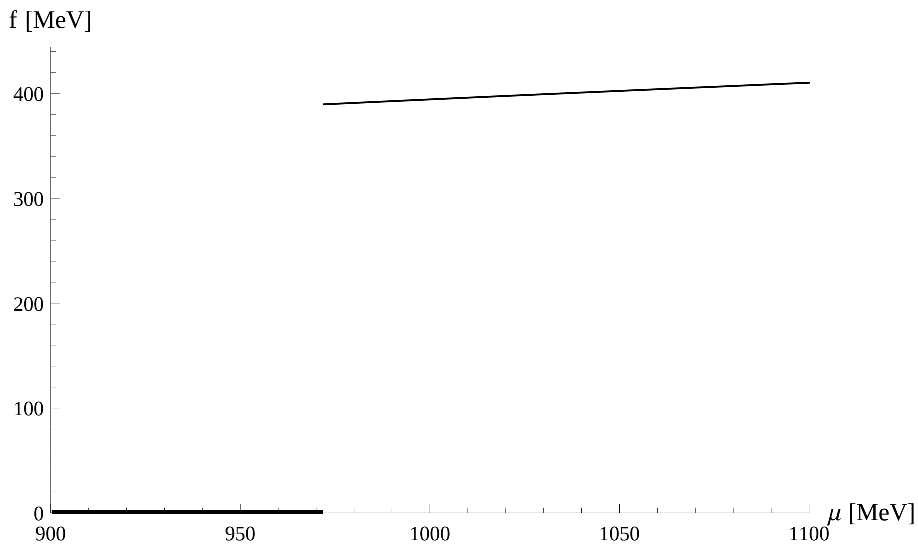


$\Longrightarrow$ **three distinct minima:**

1. the vacuum at $\phi = 154.4$ MeV (global minimum)
2. the nuclear matter ground state at $\phi = 149.5$ MeV (global minimum, degenerate with 1.) $\Longrightarrow$ first-order phase transition between 1. and 2.!
3. an inhomogeneous phase with $f \neq 0$ at $\phi = 38.3$ MeV (local minimum)

Chiral density wave in nuclear matter (III)





first-order transition to chiral density wave phase at $\mu = 973$ MeV

$\Longleftrightarrow$ **mixed phase between $\rho \simeq (2.4 - 10.4)\rho_0$**

Conclusions

I. Linear σ model with $U(N_f)_r \times U(N_f)_\ell$ symmetry with scalar, vector mesons, baryons and their chiral partners

II. Vacuum phenomenology:

1. Excellent fit of mesonic vacuum properties for $N_f = 3, 4$
2. The scalar meson puzzle: evidence for tetraquark assignment for the light scalar mesons $f_0(500)$, $f_0(980)$, glueball is most likely (predominantly) $f_0(1710)$

III. Nonzero temperature and density:

1. Chiral partners: become degenerate in mass above T_c ($f_0(1370)$ becomes lighter than $f_0(500)$ at $T_{\rm sw} < T_c$)
2. Nuclear matter ground state: correctly described by chiral effective model with mirror assignment for chiral partner of N
3. Chiral density wave in nuclear matter matter